CHAPTER 6: *Angular motion, projectile motion and fluid mechanics*

Angular motion

Angular motion is defined as **the motion of a body which twists or turns about an axis** as defined in figure 6.1. The twists, tumbles and turns involved in sports movements can all be described in this way.

Three imaginary axes of rotation

An axis of rotation is defined as '**an imaginary line about which the body rotates or spins, at right angles to the plane**' – as in figure 6.1, axes labelled A, B and C.

- **Longitudinal axis**
  - Axis A on figure 6.1.
  - This axis runs vertically from the top of the head to a point between the feet.
  - Movements in the **transverse plane** about the longitudinal axis are rotational movements.
  - Examples of sporting movements would be the spinning skater and the hammer throw (figure 6.2).
- **Transverse axis**
  - Axis B on figure 6.1. This axis runs horizontally from side to side across the body between opposite hips at right angles to the sagittal plane.
  - Movements within the **sagittal plane** about the transverse axis are flexion, extension, hyperextension, dorsiflexion and plantarflexion.
  - Sports movements about this axis include sit ups, and the high jump Fosbury Flop flight phase, and somersaults (figure 6.3).
- **Frontal axis (also** called **the sagittal axis)**
  - Axis C on figure 6.1.
  - This axis runs horizontally from front to back between belly button and lumbar spine.
  - Movements in the **frontal plane** about the frontal axis include abduction, adduction and spinal lateral flexion.
  - Examples of sports movements about this axis are a cartwheel (figure 6.4), and the bowling action in cricket.

Planes of the body

The term **body plane** is defined as '**an imaginary flat surface running through the centre of gravity of the body**', and is used to assist in the understanding of movement of body segments with respect to one another. Within each plane the turning axis can be identified as in figure 6.1.

figure 6.1 – planes and axes

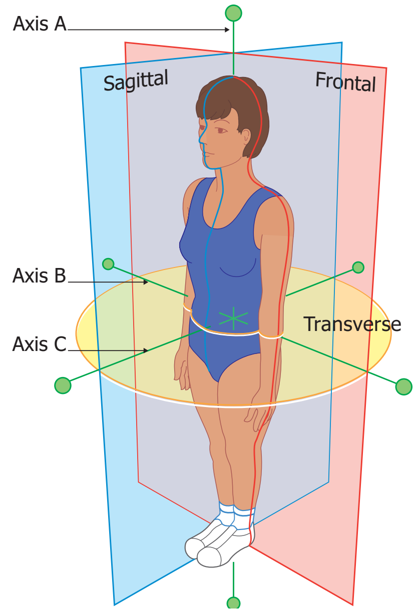


figure 6.2 – a hammer thrower turning around longitudinal axis

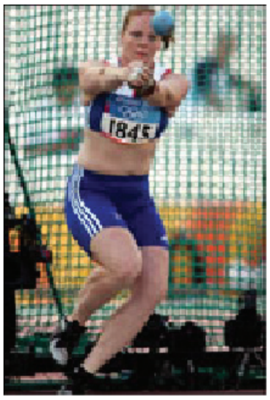
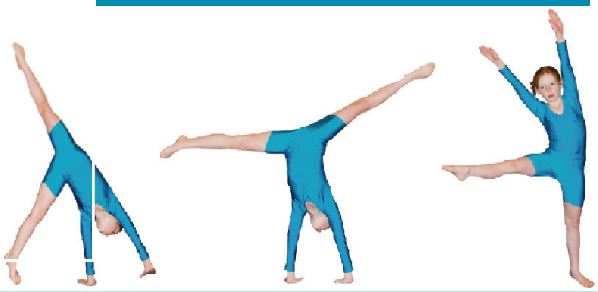


figure 6.3 – a gymnast tumbling around the transverse axis

figure 6.4 – a cartwheel rotating about the frontal axis



Angle

Angle is a familiar concept to most people, so it is readily understood what is meant by 30°, 90°, 180° and 360°. In scientific terms, an angle is measured in radians. The radian as a unit of angle is defined as '**the angle subtended at the centre of a circle by an arc length of one radius**'. Suffice it to say at this point that one radian is approximately 60°.

Angular displacement

Angular displacement is defined similarly to displacement for linear systems and is the **relative angle** compared to some fixed position or line in space. For example, if a golfer starts his/her drive from the presentation position (i.e. with club just touching the ball) and backswings to the fully extended position with the club behind his/her back, the club shaft would have an angular displacement equal to the angle between the starting position and the fully extended position of the backswing. This would be a measure of the fluency and range of the swing and could be anywhere from 180° to 290° (or 3.142 to 5.06 radians).

Angular velocity

Angular velocity is the same thing as rate of spinning or twisting, and is defined as:

$$\textbf{angular velocity } (\omega) = \frac{\textbf{angle turned (in radians)}}{\textbf{time taken to turn}}$$

This is a similar definition to that for linear velocity, except distance is replaced by angle in the formula.

Angular acceleration

Again in a similar way to linear systems, it is possible to define angular acceleration as:

$$\textbf{angular acceleration} = \frac{\textbf{change of angular velocity}}{\textbf{time taken to change}}$$

This concept applies to situations in which the rate of spin **changes with time**. Examples of this would be the hammer throw (in which the rate of spin increases throughout the movement up to the release of the hammer) and the tumbler, gymnast or diver (who speeds up the rate of rotation or slows it down by changing his/her body shape).

Torque

Torque is the twisting force which you could apply to a body to cause it to turn or spin. It is defined as the force applied to the body multiplied by the perpendicular distance to the axis of rotation (the moment of force about the turning axis).

This definition means that the bigger the force and the distance from the axis of turning, the bigger the turning effect.

Moment of inertia (MI)

This is the equivalent of mass (**inertia**) in the linear system, and is defined as:

moment of inertia = sum of [(mass of body part)
x (distance of body part from the axis of rotation) squared]
over all parts of the rotating body.

Mathematically: $MI = \Sigma m r^2$

Objects rotating with large MI require large moments of force (torque) to change their angular velocity, and objects with small MI require small moments of force (torque) to change their angular velocity or ω.

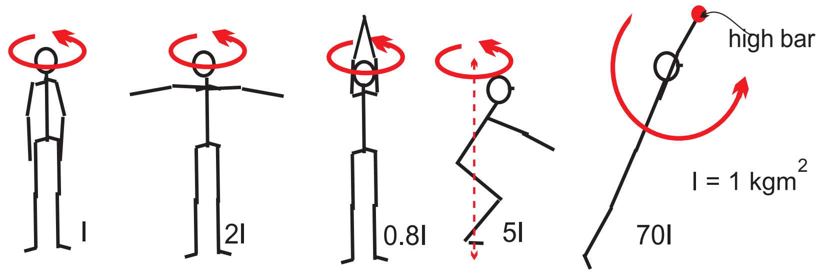
Moment of inertia

The formula on page 88 means that moment of inertia depends on the spread of mass away from the axis of spin, so as the body shape changes, the moment of inertia of the shape changes. The more spread out the mass, the bigger the MI.

The unit of MI is kilogramme metre squared - kgm^2.

- Bodies with **arms held out wide** have large MI, the further the mass is away from the axis of rotation increases the MI dramatically.
- Sportspeople use this to control all spinning or turning movements.
- Pikes and tucks are good examples of use of MI, both reduce MI.

figure 6.5 – moments of inertia of different shapes



Values of moment of inertia

In figure 6.5, I is the MI for the left most pin man and has a value of about 1.0 kgm^2 for an average male person. From this diagram you can see how control of the arms will make a big difference to the value of MI, and that a tuck or pike can also **change MI** dramatically.

Angular momentum

Angular momentum is a quantity used to describe what happens when bodies spin and turn, it is defined as:

angular momentum = **moment of inertia** x **angular velocity**
= **rotational inertia** x **rate of spin**
H = **I** x ω

Conservation of angular momentum

The **law of conservation of angular momentum** is a law of the universe which says that angular momentum of a spinning body remains the same (provided no external forces act)

figure 6.6 – moment of inertia is very large with both arms and legs spread wide

Conservation of angular momentum

- This means that a body which is spinning, twisting or tumbling will keep its value of **H** once the movement has started.
- Therefore if moment of inertia (**I**) changes by changing body shape, then angular velocity (ω) must also change to keep angular momentum (**H**) the same.
- So, if MI (**I**) **increases** (body spread out more) then ω must **decrease** (rate of spin gets less).
- And conversely, if MI (**I**) **decreases** (body tucked in more) then ω must **increase** (rate of spin gets bigger).
- Strictly, this is only exactly true if the body has no contact with its surroundings, as for example a high diver doing piked or tucked somersaults in the air, but it is almost true for the spinning skater!

figure 6.7 – a spinning skater

Sporting examples of conservation of angular momentum

- **The spinning skater**. If the arms are wide, the MI is large and the skater spins slowly. If the arms are brought in, MI is small and the skater will spin more quickly (figure 6.7).
- **The tumbling gymnast** (figure 6.8). With the body position open, the MI is large and the gymnast (or diver or trampolinist) will spin slowly. When he or she creates a tucked body position, the MI is small and he or she will spin more quickly.
- **The dancer doing a spin jump** (figure 6.9). The movement is initiated with arms held wide which would therefore have the highest possible MI. Immediately he or she has taken off, the angular momentum is conserved, and so by tucking the arms across the chest, this will create the lowest possible MI. This then means that he or she will acquire the highest possible rate of spin, so that more spins can be completed before landing.
- **The slalom skier**. The slalom skier crouches on approach to the gate and therefore will have a large turning MI. As he or she passes the gate, he or she stands straight up (reducing MI). This enables the person to turn rapidly past the gate, then he or she crouches again (figure 6.10) - increasing MI which will resume a slow turn between the gates.

figure 6.8 – a tumbling gymnast

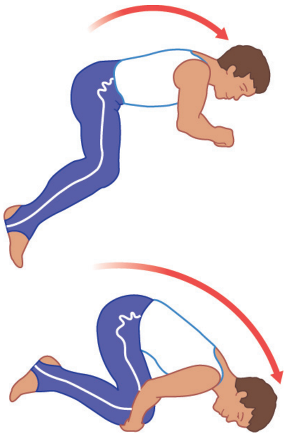
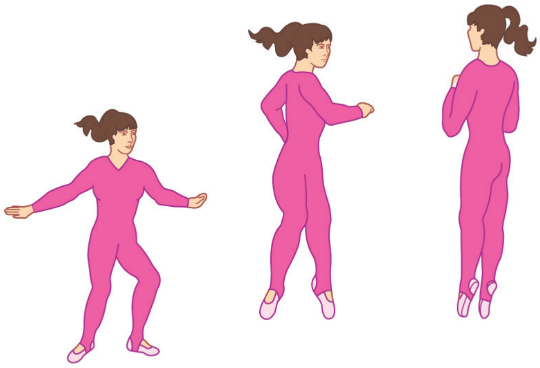


figure 6.9 – a spinning dancer



Newton's laws of angular motion

The laws of angular motion are similar to Newton's laws of linear motion except that they apply to turning, spinning, or twisting system or bodies. They are:

- **Newton's first law of angular motion** states that a spinning or rotating body will continue with a constant angular momentum except when acted upon by an external force or torque.
- **Newton's second law of angular motion** states that the rate of change of angular momentum of the body will be proportional to the torque acting on it, and in the same direction (of spin) as the torque.
- **Newton's third law of angular motion** states that when a torque is applied to one body by another body, an equal but opposite in direction torque will be applied by the second body to the first body.

figure 6.10 – a slalom skier

Projectile motion

Factors affecting horizontal displacement of projectiles

This section looks at the **motion of objects in flight**, such as human bodies (during the flight phase of a jump), throwing implements (shot, discus, javelin or hammer), and soccer, rugby, cricket, tennis and golf balls.

The flight is governed by the forces acting, the weight, air resistance, Magnus effect (page 95), aerodynamic lift, and the direction of motion. If weight were the only force acting, the flight path would be **parabolic** in shape, and some flight paths are similar to this (shot or hammer, the human body in jumps or tumbles or dives as in figures 6.11, 6.12, and 6.13, where weight is the predominant force acting).

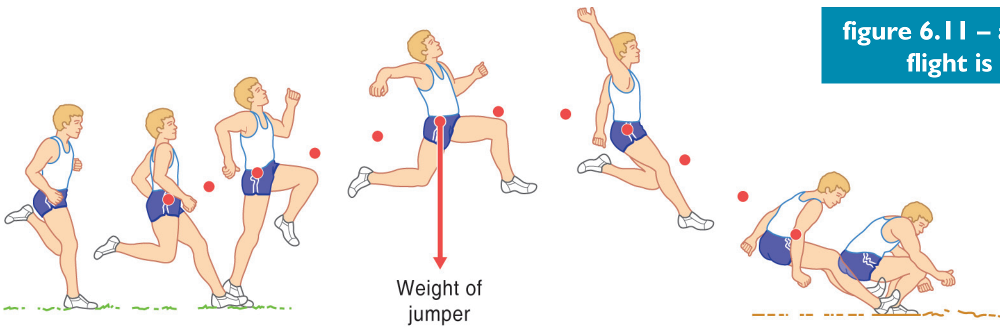


figure 6.11 – after take-off, a long jumper's flight is governed by weight only

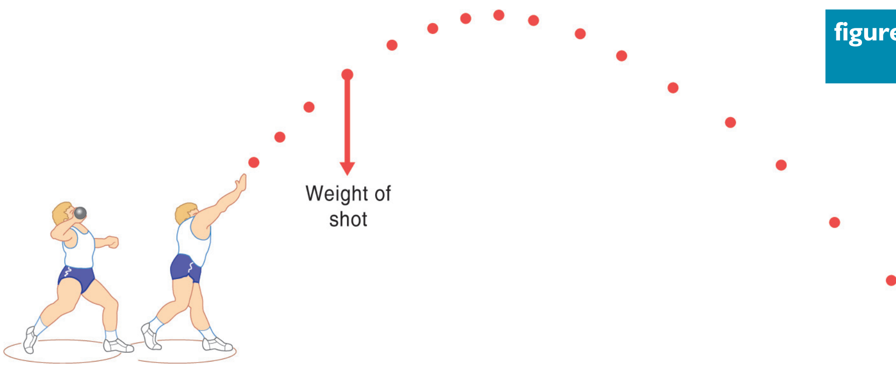


figure 6.12 – after release, the flight of a shot is governed by gravity only

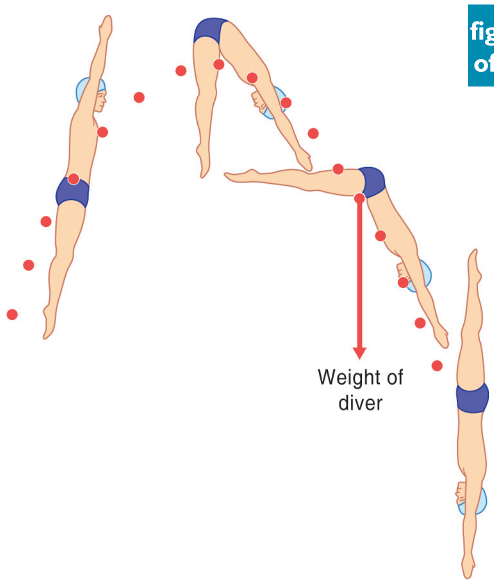


figure 6.13 – after leaving the board, the flight of a diver is governed by gravity (weight) only

Factors affecting horizontal displacement of projectiles

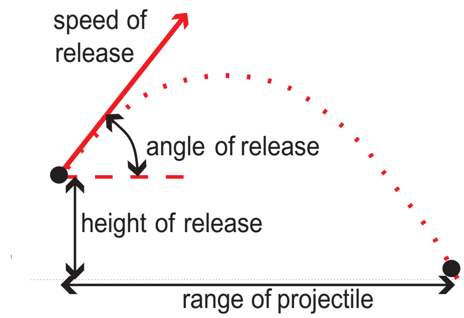
Figure 6.14 summarises the factors which influence the distance travelled, the **angle of release**, the **speed of release**, and the **height of release**.

The optimum distance moved before landing is acheived at 45° release angle.

If the height of release is about 2 metres off the ground, as in the shot put (figure 6.14), then the optimum angle of release (to achieve maximum distance) will be less than 45°, probably approximately 42°.

But if the landing of the object thrown is higher than the point of release (as in the case of a basketball shot), then the optimum angle of release will be greater than 45°.

figure 6.14 – factors affecting distance travelled



The relative size of forces during flight

The forces acting during flight are: the weight of the object, the air resistance or drag, (the faster the projectile travels the greater will be the air resistance), **aerodynamic lift**, and the **Bernoulli effect** (page 94) or **Magnus effect** (page 95).

If the shapes of the flight path differ from a parabola then some combination of these forces must be relatively large compared with the weight (remembering that a flight of an object with only weight force acting would be a parabola).

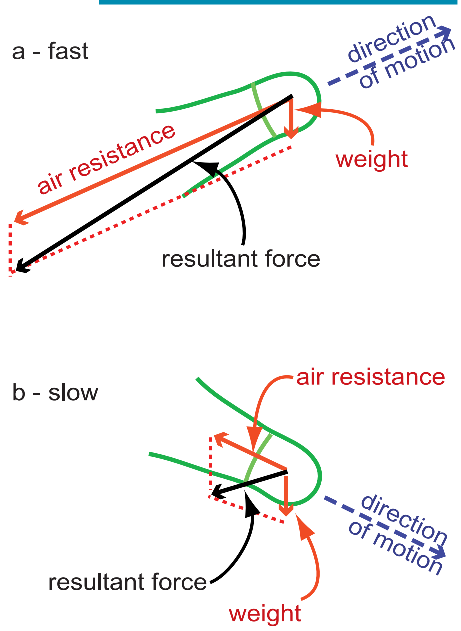
For example, the badminton shuttle

For a badminton shuttle **struck hard** (figure 6.15a), the air resistance is very large compared with the weight, because the shuttle is moving quickly. The resultant force will therefore be very close to the air resistance. This would make the shuttle slow down rapidly over the **first part of the flight**.

Later in the flight of the badminton shuttle (figure 6.15b), when the shuttle is moving much more slowly, the air resistance is much less and comparable with the weight. This pattern of the resultant force changing markedly during the flight predicts a pronounced asymmetric path.

Figure 6.16 shows a badminton shuttle's flight path, which is markedly asymmetric, because of the change of predominant force during the flight.

figure 6.15 – forces on a badminton shuttle



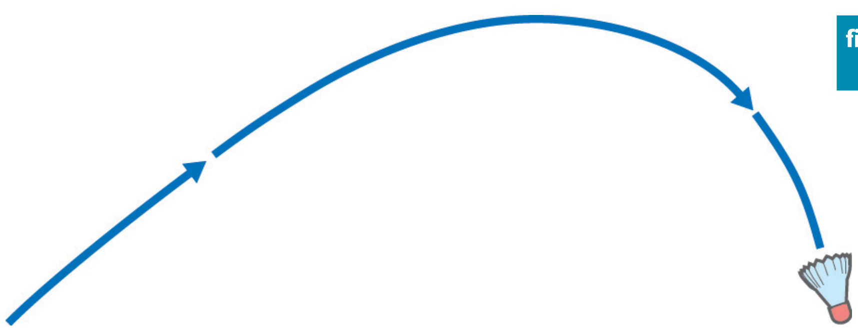


figure 6.16 – asymmetric flight of a badminton shuttle

Vector components of velocity during flight

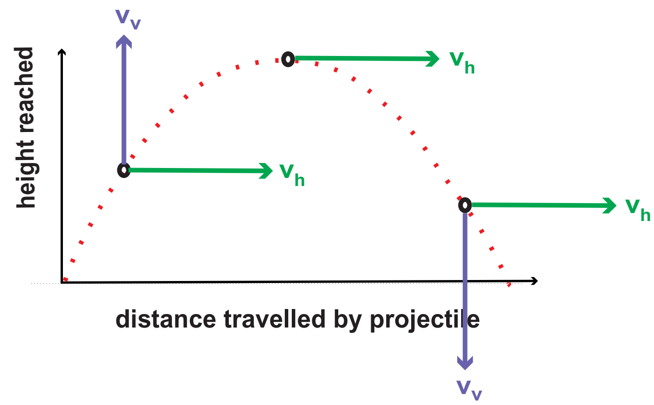
Figure 6.17 outlines the **vertical** and **horizontal components** of the velocity during the flight of a projectile.

Near the start of the flight:
- There is a large **upward vertical component** v_v.
- But a **fixed horizontal component** v_h.

At the middle of the flight:
- There is zero **upward vertical component**, since the object moves entirely horizontally at this point.
- There is still a **fixed horizontal component** v_h, which is the same as at the start.

Near the end of the flight:
- There is a large **downward vertical component** v_v, which is almost the same as the **horizontal component** since the object is travelling at approx 45° to the horizontal downwards.
- There is still a **fixed horizontal component** v_h, which is the same as at the start.

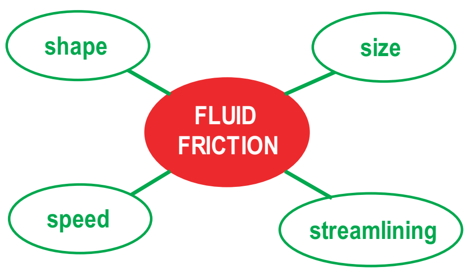
figure 6.17 – components of velocity during flight



Fluid mechanics

Fluid friction force depends on the shape and size of the moving object, the **speed** of the moving object, and the streamlining effect (summarised in figure 6.18).

figure 6.18 – factors affecting fluid friction or air resistance



Drag and air resistance

In order to minimise drag, the following developments affect sport:
- The body position and shape for a swimmer.
- The shape of helmets for cyclists.
- The use of lycra clothing.
- The shape of sports vehicles (cars or bikes).

Low values of fluid friction

This discussion concerns **low values of drag** compared with other forces. Examples are:
- Any sprinter or game player for whom air resistance is usually much less than friction effects and weight. Therefore streamlining is seen as less important.
- A shot or hammer in flight, in which air resistance would be much less than the weight, and therefore the angle of release should be around 45°.

High values of fluid friction

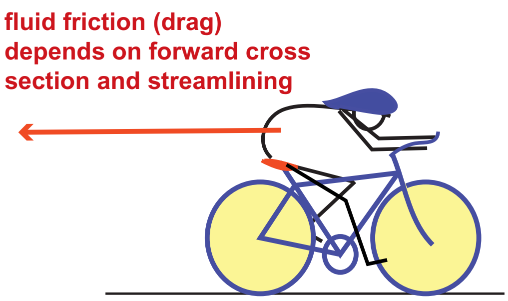
High values of drag will occur for any sportsperson or vehicle moving through water, and hence fluid friction is the critical factor governing swimming speed.

- Body shape or cross section, and clothing (surface material to assist laminar flow, on page 94), are adjusted to minimise fluid friction.

A cyclist (figure 6.19) travels much faster than a runner and therefore has **high fluid friction**:
- He or she crouches low to reduce the forward cross sectional area.
- The helmet is designed to minimise turbulent flow.
- Clothing and wheel profiles are designed to assist streamlining.

Cross sectional area is the area of the moving object as viewed from the front. The smaller the better to reduce drag, hence cyclists crouch down, and keep their elbows in.

figure 6.19 – a cyclist needs good streamlining

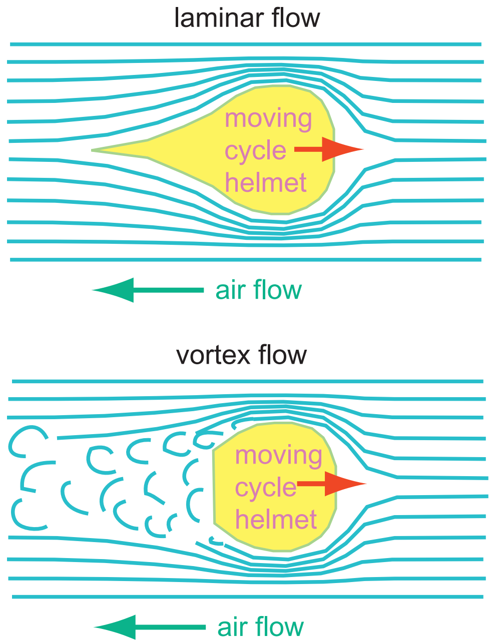


Laminar flow and drag

Fluid friction (or drag) depends on **laminar** flow, the smooth flowing of air or water past an object. Laminar means flowing in layers, and streamlining assists laminar flow. Figure 6.20 shows images of a streamlined helmet, and a non-streamlined helmet. The point of the streamlined shape is that the air moves past it in layers whereas in the case of the non-streamlined helmet, vortices are formed where the fluid does not flow smoothly. When this happens bits of fluid are flung randomly sideways which causes drag.

The drag is caused by bits of fluid being dragged along with the moving object (the cycle helmet).

figure 6.20 – laminar flow and vortex flow



Lift force

Dynamic **lift** (upward force) can be caused by the movement of the body. As the body moves forward, the angle presented by the lower surface of the body to the direction of motion (called the angle of attack) can cause the air molecules through which the object is moving to be deflected downward and hence would cause a downward force on the air through which the object passes (figure 6.21).

This **downward force on the air** would cause an **upward force on the moving object** in reaction to the downward force on the air (by Newton's third law). This is the lift.

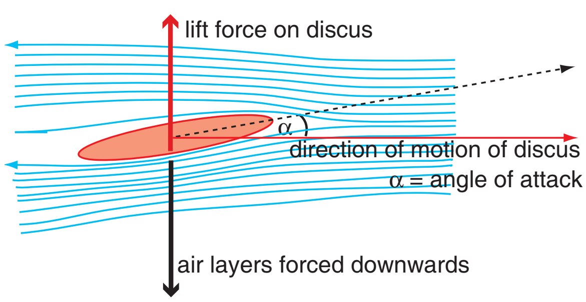
Such a force can explain the flight of a discus.

A discus is a symmetrical object, which would therefore not be subject to the Bernoulli force which explains the flight of a wing moving horizontally through air.

The angle of attack of the discus is such as to present its lower surface to the air flow which causes the lift as explained above.

There is a distinction between a force caused as a reaction to air (or water) thrown up or sideways to the direction of motion of for example a downhill skier or a cyclist.

figure 6.21 – lift force on a discus



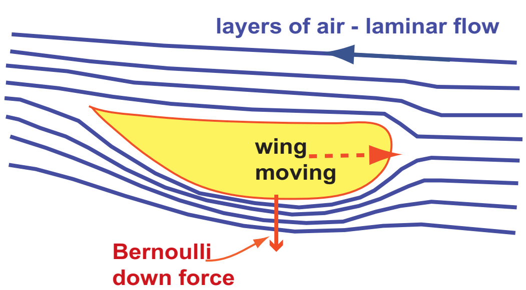
The Bernoulli effect

The force which gives lift to aircraft wings, and down-pressure on racing car bodies (figure 6.22, enabling greater friction between wheels and the road, and hence faster cornering speeds) is called the **Bernoulli effect**.

This effect depends on the fact that fluids which move quickly across the surface of an object cause a reduced pressure when compared with slower moving air across another surface.

Hence, in figure 6.22, the laminar flow of air across the **lower** surface of the wing (or car body shaped like an inverted wing) is **quicker**, because the air has to travel **further** in the same time as the air moving a shorter distance across the upper surface of the wing. Hence the shape of the wing is crucial to create the Bernoulli lift (in aeroplanes) or down force (in racing cars, figure 6.23 page 95).

figure 6.22 – Bernoulli effect on an inverted wing



The Bernoulli effect

The **Bernoulli effect** has been built into racing cars to increase the down force (which would therefore increase the friction force between wheels and the ground).

So, formula I racing car manufacturers build this **shape into the whole car** (figure 6.23), not just the artificial wings sometimes attached to the car upper surfaces.

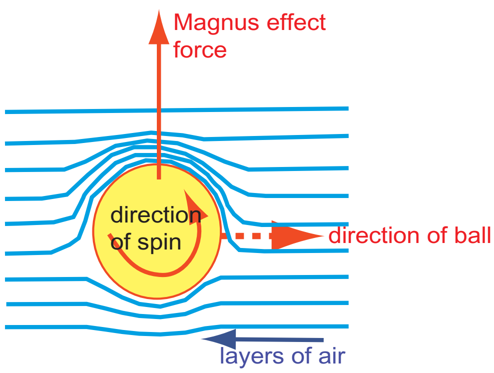
figure 6.23 – shape of a racing car body

The Magnus effect

The **Magnus effect** is the Bernoulli principle applied to spinning balls.

As a **spinning ball** moves through the air (from left to right on figure 6.24), the air layers which flow round the ball are forced into the path shown in the diagram. Here you can see that the air flow is **further** round the top of the ball, and hence the air flow is **faster** over the top of the ball than the bottom. This means that the **air pressure** will be **less** over the top of the ball than the lower half of the ball (following from the Bernoulli effect), hence the ball will experience a force upwards in the view of figure 6.24.

figure 6.24 – Magnus effect on a spinning ball



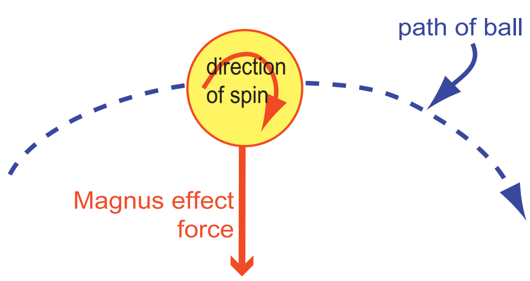
Hence **top-spin** as shown in figure 6.25, would cause a dipping effect on the ball in flight, the force is downward in this figure.

Similarly, **side-spin** will cause a swerve in the flight whose direction is in the same sense as the spin of the ball.

Golfers (figure 6.27, page 96) cause a ball to fade to the right or hook to the left by imparting side-spin to the ball during the strike.

The diagram in figure 6.26a show how side spin causes swerving sideways by the golfers.

figure 6.25 – flight of a spinning ball



The sense of swerve is in the same direction as the spin on the ball.

The sports in which back spin (figure 6.26b) and top spin (figure 6.26c) are used to the maximum are raquet sports such as tennis and table tennis.

Most tennis players use the top spin effect to cause a ball to dip into the opponent's court after a very firm hit of the ball. Rafael Nadal is a prime exponent of this technique.

figure 6.26 – flight of a spinning ball

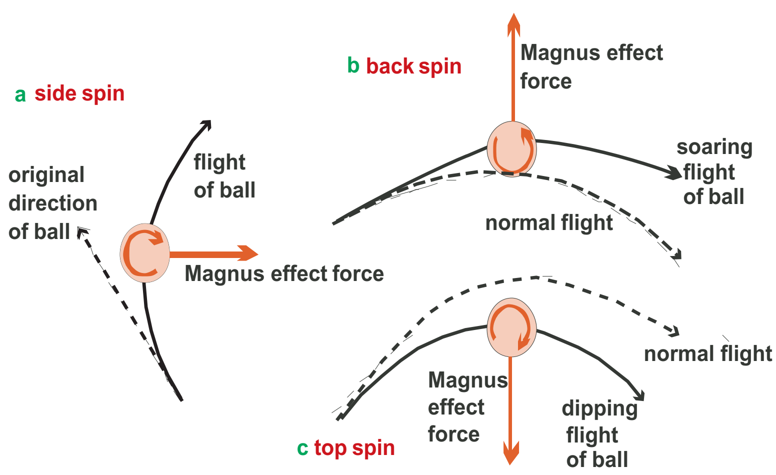


figure 6.27 – golfers can control ball spin to place the ball on the green

Practice questions

1) Angle in radians is defined as:
   a. rate of turning.
   b. arc length subtending the angle divided by radius of the circle.
   c. radius of a circle divided by arc length subtending the angle.
   d. moment of inertia divided by angular velocity.

2) Angular velocity is defined as:
   a. angular acceleration divided by time taken to turn through an angle.
   b. distance moved per second in a certain direction.
   c. angle turned through in radians divided by time taken to turn
   d. moment of inertia divided by angular momentum.

3) Which sentence best explains the flight of a projectile?
   a. the projectile travels further if air resistance is large compared with its weight.
   b. a projectile ejected at 45° to the horizontal will travel the furthest.
   c. the flight path of a projectile falls from its initial direction caused by gravity only..
   d. weight and fluid friction are the only forces acting on a projectile.

4) The Bernoulli effect causes a sideways force on an object moving through a fluid because:
   a. fluids flow in a laminar pattern past a moving object.
   b. the pressure exerted by a fast moving fluid is less than that exerted by a slow moving fluid.
   c. the pressure exerted by a fast moving fluid is greater than that exerted by a slow moving fluid.
   d. an unstreamlined object will cause fluid flow to break into vortices.

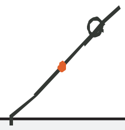
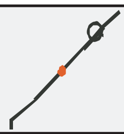
5) A dancer spinning with arms out wide will spin slower than when he crosses his arms across his chest during a jump because:
   a. angular momentum is bigger with his arms out wide.
   b. angular momentum is smaller with his arms across his chest.
   c. moment of inertia of his body with his arms out wide is bigger.
   d. angular momentum is conserved during the flight of the dancer.

6) Define the term angular velocity. 2 marks

Practice questions

7) a) A diver can make a number of different shapes in the air. Table 6.1 shows three of these. Explain the meaning of moment of inertia (MI) in this context. 4 marks

Table 6.1 – **data for shapes of diver during flight**

phase of dive	shape of diver	time during flight	MI of shape kgm^2
1	Z 	0.0 - 0.5s	18
2	Y 	0.5 - 0.7s	9
3	X 	0.7 - 1.0s	3
4	Z 	1.0 - 1.1s	18
entry	axis of rotation = ●	1.1s	

b) During a dive a diver goes through the shapes shown in table 6.1. Explain how the rate of spinning (angular velocity) would change through the dive. 5 marks

c) Sketch a graph of this rate of spinning against time. Your sketch need only be approximate. 4 marks

d) State the relationship between angular momentum, moment of inertia and angular velocity. 2 marks

e) Name the law of conservation which accounts for these variations in rate of spin. 1 mark

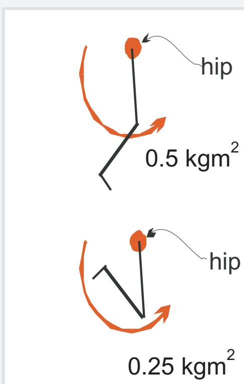
f) Explain and sketch the arc described by the diver as he or she falls. 3 marks

8) a) Describe in detail the body shape and movement within a chosen sporting situation where rates of spin are affected by body shape. 6 marks

b) How would you stop the spinning in this situation? 2 marks

c) Figure 6.28 shows a sportsperson's leg in two different positions. The values quoted are the moment of inertia of the leg as it rotates about the hip joint (shown as a red dot on each diagram). Explain the implications of these data for the efficiency of running style in a sprinter and long distance runner 7 marks

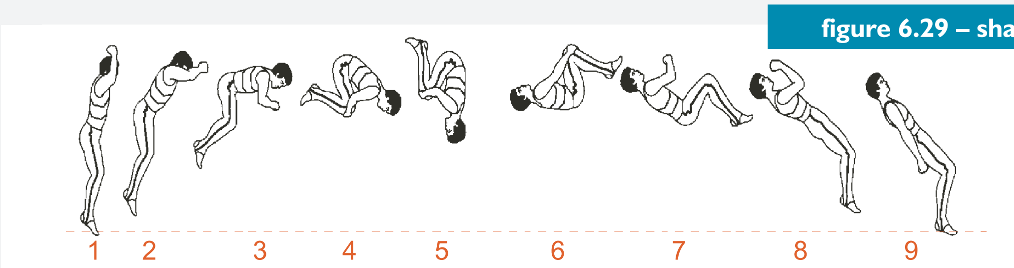
figure 6.28 – shape of leg



Practice questions

9) a) Figure 6.29 shows a gymnast undertaking a forward somersault following a run up. Sketch three traces on a single graph to represent any changes in angular momentum, moment of inertia and angular velocity for the period of activity between positions 2 and 9. 3 marks

figure 6.29 – shapes of a gymnast



b) Explain the shapes of the traces on the sketch graph that you have drawn. 6 marks

c) Table 6.2 sets out measurements of angular velocities (rates of spin) of the gymnast at successive frames from the start of the somersault.

Estimate from this table the ratio of angular velocities at times X and Y. 1 mark

d) If the moment of inertia of the gymnast is 8 kgm^2 at time X, estimate the moment of inertia at time Y, using data from table 6.2. 2 marks

Table 6.2 – **data for angular velocity of gymnast**

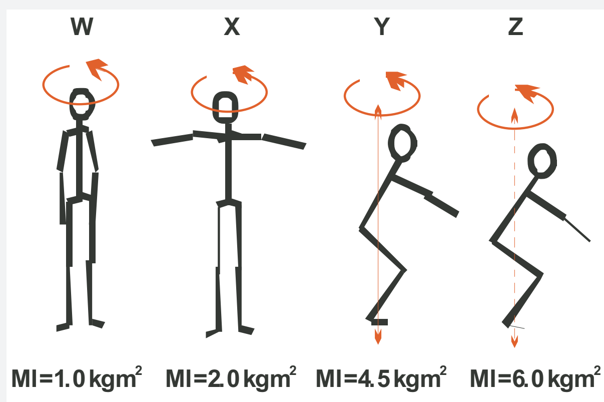
	frame	angular velocity (degrees s^{-1})
	1	650
X	2	750
	3	850
	4	1100
	5	1400
Y	6	1500
	7	1000
	8	850
	9	650

10) a) Figure 6.30 shows a spinning skater in various positions. Under each diagram is an approximate value for the moment of inertia of the skater spinning about his or her central vertical axis.

The angular velocity of the skater in position **W** is 2.0 revolutions per second. What is the formula for calculating the skater's angular velocity?

Calculate the angular velocity for the skater in position **Z**. 2 marks

figure 6.30 – shapes of a skater



b) Sketch a figure showing a possible position which could cause the skater to attain an angular velocity of 3.0 revolutions per second and calculate what the moment of inertia of this shape must be. 2 marks

c) Principles of angular momentum can be used to improve performance in a variety of sports. With the use of diagrams explain how a slalom skier turns through the gates at maximum speed. 4 marks

d) Explain with the use of diagrams how a dancer manages to complete a triple spin in the air before touching the ground. 4 marks

Practice questions

11) a) Using examples, explain how the shape of an object can alter its flight path. 4 marks

b) Explain the effect of air resistance on the flight of two badminton shuttles, one of which has been struck hard and the other gently. 10 marks

c) Briefly explain why the flight path of a shot in athletics is so different from the flight of a badminton shuttle. 4 marks

12) a) Identify three physical factors (not skill factors) which govern a swimmer's speed and explain how one of these occurs. 3 marks

b) Describe the factors which determine the amount of fluid friction acting on a swimmer. 4 marks

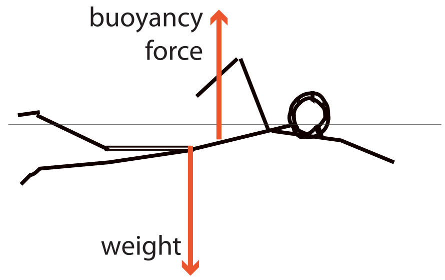
c) Explain how you would minimise turbulent flow (high drag) of the water past the swimmer's body. 2 marks

d) Give three examples, each from a different sporting context, to show how fluid friction affects the sportsperson. 3 marks

e) How would you attempt to reduce fluid friction? 3 marks

f) Look at figure 6.31 showing the vertical forces acting on a swimmer during a stroke.
Explain why it is difficult for a swimmer to keep a horizontal floating position. 4 marks

figure 6.31 – forces acting on a swimmer



13) a) Fluid friction is a force which acts on a bobsleigh once it is moving. Identify the nature of the fluid friction in this case and explain how this might limit the maximum speed of the bob. 3 marks

b) Explain the term 'turbulent flow', and how the bobsleigh is used to minimise this factor. 3 marks

14) a) Sketch a diagram to show the flight path of the shot from the moment it leaves the putter's hand to the moment it lands. 2 marks

b) State and briefly explain three factors (excluding air effects) which should be used by the putter to optimise the distance thrown. 6 marks

c) Explain why the turn in a discus throw produces greater horizontal range than the standing throw. 3 marks

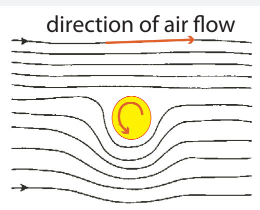
15) a) The Bernoulli effect states that a faster flowing liquid or gas exerts less pressure than a slower moving liquid or gas. Using figure 6.32, show how the Bernoulli effect explains the swerve of a spinning ball. 4 marks

b) Use diagrams to show how your explanation relates to the flight of a table tennis ball with side, back and top spin. 3 marks

c) Sketch a vector diagram of all forces acting on a table tennis ball in flight with back spin, and explain how the resultant force on the ball predicts the actual acceleration of the ball. 4 marks

d) Identify one sport other than a ball game, in which the Bernoulli effect plays a part. 1 mark

figure 6.32 – Bernoulli effect on a spinning ball



Answers link: http://www.jroscoe.co.uk/downloads/a2_revise_pe_aqa/AQAA2_ch6_answers.pdf